AP Calculus BC
Lesson 8.7 Approximate Integration (Left, Right, Midpoint, Trapezoid Rules)

1. (a) Write a Riemann Sum formula that will calculate the right sum for any continuous function *f*, defined on [*a*,*b*].

(b) Write a Riemann Sum formula that will calculate the left sum for any continuous function *f*, defined on [*a*,*b*].

(c) Write a Riemann Sum formula that will calculate the midpoint sum for any continuous function *f*, defined on [*a*,*b*].

(d) Write a Riemann Sum formula that will calculate the trapezoidal sum for any continuous function *f*, defined on [*a*,*b*].

(e) Use your formulae from above with $n = 4$ to approximate $\int_0^4 \sqrt{1+x}\,dx$.

(f) For the data given in the table below, use the trapezoidal sum with $n = 7$ to approximate the area under the function *f*, from 1 to 8.

x	1	2	3	4	5	6	7	8
$f(x)$	5.6	-1.4	5.8	11.2	12.5	6.3	4.1	0

2. Use a left sum, a right sum, the trapezoidal rule and the midpoint rule with $n = 20$ to find approximating sums for:

(a) $\int_0^6 e^{\sqrt{x}}dx$

(b) $\int_1^5 \sqrt{1+x^4}dx$

(c) $\int_0^5 \left(x^2 - 4x\right)dx$.

3. Suppose that f is **monotone** on $[a,b]$, and let $I = \int_a^b f(x)dx$. Then:

$$\left|I - R_n\right| \le \left|f(b) - f(a)\right| \cdot \frac{(b-a)}{n};$$

$$\left|I - L_n\right| \le \left|f(b) - f(a)\right| \cdot \frac{(b-a)}{n};$$

Why?

For each sum, find an upper bound on the error in the approximation.

(a) $\int_0^5 (x^2 + 4x + 3)dx$; use left sums with $n = 25$.

(b) $\int_0^{\pi/2} \sin(x)\, \mathrm{d}x$; use right sums with $n = 25$.

4. Let $I = \int_0^1 f(x)dx$ and suppose that f is decreasing on $[0,1]$ such that $f(0) = 7$ and $f(1) = 4$ and $L_{16} = 5.3172$.

(a) Does L_{16} underestimate I? Justify your answer.

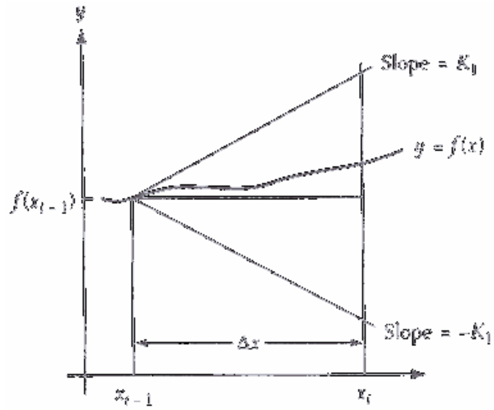
(b) Find an upper bound on $\left|I - L_{16}\right|$.

(c) Evaluate R_{16}.

5. (a) Use the diagram below to explain why the left error bound is valid:

$$\left|I - L_n\right| \le \frac{K_1(b-a)^2}{2n} \text{ where } \left|f'(x)\right| \le K_1 \text{ on } [a,b].$$

Left rule error over one subdivision



(b) Use this error bound to estimate the error in using either L_{10} or R_{10} to estimate the following integrals. Finally check to see that the error estimate works by actually calculating the approximations and the actual value of the integral.

$$\int_0^2 \sin(x)dx$$

$$\int_0^4 (x^2 - 4)dx$$

(c) Estimate the maximum amount of error that could result from using L_{10} or R_{10} to estimate the following integrals.

$$\int_0^3 e^{-x^2} dx$$

$$\int_0^5 e^{\sin(x)} dx$$

6. On page 557 of your text are upper bounds on the error for both the midpoint and trapezoid sums. These are:

For Midpoint Sums: $\left|I - M_n\right| \le \frac{K_2 (b-a)^3}{24n^2}$

and

For Trapezoid Sums: $\left|I - T_n\right| \le \frac{K_2 (b-a)^3}{12n^2}$

where $K_2 \ge \left|f''(x)\right|$ for all x on $[a,b]$

Notice that both the trapezoid sum and midpoint sums commit no error if the function f is linear. (Why does this work for midpoint as well as trapezoid?) Hence, the error must be a function of the second derivative. It is the concavity that causes the error in these sums.

Estimate the error involved in computing the following integrals with the trapezoid sum and the midpoint sum with n = 100 using error analysis for each of the following.

(a) $\int_0^8 \sin(x)dx$

(b) $\int_2^5 e^{-x}dx$

(c) $\int_1^6 \sqrt{1+x}dx$

7. Find a value of n for the trapezoid sum and a value of n for the midpoint sum that will give approximations accurate to the nearest thousandth for each of the following:

(a) $\int_2^5 \cos(x)dx$

(b) $\int_{-1}^4 e^{-x}dx$

(c) $\int_1^3 e^{-x^2}dx$

(d) $\int_1^4 \sqrt{1+2x}dx$