AP Calculus BC
Lesson 8.7 continued: Simpson's Rule

1. Simpson's rule divides the interval into an even number of subintervals and on each pair of subintervals approximates the area under the curve using a parabola on top. Often this produces a much better approximation than the trapezoid rule.

   (a) Consider $\int_0^{\pi} \sin(x)dx$.

   Divide the interval from 0 to π into two subintervals $\left[0,\frac{\pi}{2}\right]$ and $\left[\frac{\pi}{2},\pi\right]$. This gives three points on the graph of the sine function, namely, (0, 0), $\left(\frac{\pi}{2},1\right)$, and ($\pi$, 0).

   Find an equation of the parabola with equation of the form $y = ax^2 + bx + c$ that passes through these three points. Find the area under the parabola with a definite integral.

2. Consider the parabola defined by $y = ax^2 + bx + c$ on the closed interval [$-h$, h].

   (i) Show that $\int_{-h}^{h}\left(ax^2+bx+c\right)dx=\frac{h}{3}\left(2ah^2+6c\right)$.

   (ii) If the parabola passes through the points ($-h$, y_0), (0, y_1), and (h, y_2), show that $y_0 = ah^2 - bh + c$, $y_1 = c$, and $y_2 = ah^2 + bh + c$.

   (iii) Use parts (i) and (ii) to show that $\int_{-h}^{h}\left(ax^2+bx+c\right)dx=\frac{h}{3}\left(y_0+4y_1+y_2\right)$.

   (iv) Show that the integral $\int_a^b f(x)\,dx$ can be approximated by

   $S_n=\frac{h}{3}\left(y_0+4y_1+2y_2+4y_3+2y_4+\cdots+2y_{n-2}+4y_{n-1}+y_n\right)$, where [$a$, b] has been partitioned into an even number, n, of subintervals of length $\Delta x=h=\frac{b-a}{n}$.

   (Note: The y's in the equation are the values of $y = f(x)$ at the partition points $x_0 = a$, $x_1 = a + h$, $x_2 = a + 2h$, ... , $x_{n-1} = a + (n - 1)h$, and $x_n = b$.)

3. (a) Use Simpson's rule with $n = 2$ to approximate $\int_0^{\pi} \sin(x)dx$.

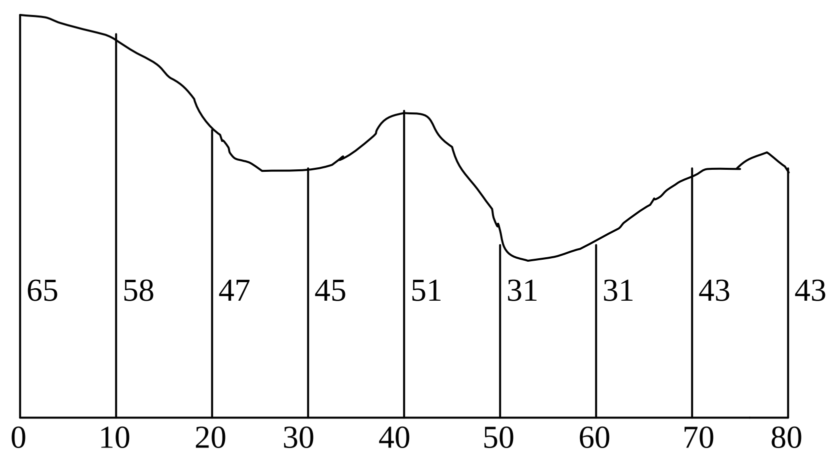
Compare your results with your answer to question 1.

(b) Use Simpson's rule with $n = 30$ to approximate $\int_0^{\pi} \sin(x)dx$.

(c) Use the trapezoidal rule with $n = 2$ to approximate $\int_0^{\pi} \sin(x)dx$.

(d) Which approximation seems the best, especially when contrasted with the exact value of $\int_0^{\pi} \sin(x)dx$?

4. The drawing below represents a parcel of property running along the jagged side of a river. The owner measures at each 10-meter interval on the straight edge of her property from that edge to the bank of the river. Using the measurements as recorded above (all measurements in meters), approximate the area of her property using



(a) Simpson's rule with $n = 2$,

(b) Simpson's rule with $n = 8$, and

(c) the trapezoidal rule with $n = 8$..

5. Let $I = \int_a^b f(x)\,dx$, and let K_4 be an upper bound for $\left|f^{(4)}(x)\right|$ on $[a,b]$. Then $\left|I - S_n\right| \le \dfrac{K_4 (b-a)^5}{180n^4}$. It is beyond the scope of this course to prove this result.

(a) If Simpson's rule is used with 20 subintervals, how close do we know our Simpson's rule approximation for $\int_0^{\pi} \sin(x)\,dx$ to be to the exact value of this integral?

(b) If we need to have a Simpson's rule approximation for $\int_1^4 \ln(x)\,dx$ to be accurate within 0.01 of the actual value, how many subintervals do we need?