

AP Calculus BC  
Lesson 9.1 Arc Length

9.1(1) Consider  $y = f(x) = x^2$  for  $0 \leq x \leq 2$ .

- (a) Approximate the length of this curve by using line segments from (0,0) to (1,1) and (1,1) to (2,4).
- (b) If necessary, rewrite your expression in the form  $k(\sqrt{1+a} + \sqrt{1+b})$ . Try to determine the relationship (if any exists) among  $a$ ,  $b$ , and the slopes of the line segments.
- (c) Repeat part a with four equal line segments: (0,0) to  $\left(\frac{1}{2}, \frac{1}{4}\right)$ ,  $\left(\frac{1}{2}, \frac{1}{4}\right)$  to (1,1), etc.
- (d) Rewrite your expression in the form  $k(\sqrt{1+a} + \sqrt{1+b} + \sqrt{1+c} + \sqrt{1+d})$ . Determine any geometric significance of  $k$ ,  $a$ ,  $b$ ,  $c$ , and  $d$ .
- (e) Make a conjecture about how calculus can be used to find the **exact** arclength from (0,0) to (2,4).
- (f) Approximate this arclength.

9.1(2) Find the length of the semicubical parabola  $y^2 = x^3$  between the points (1,1) and (4,8).

9.1(3) Find the length of the arc of the curve  $8y = x^4 + 2x^{-2}$  from the point where  $x = 1$  to the point where  $x = 2$ .

9.1(4) Find the length of the arc of the curve  $6xy = y^4 + 3$  from the point where  $y = 1$  to the point where  $y = 2$ .