RC Circuits

"The concept is interesting and well-formed, but in order to earn better than a 'C,' the idea must be feasible." A Yale University management professor in response to Fred Smith's paper proposing

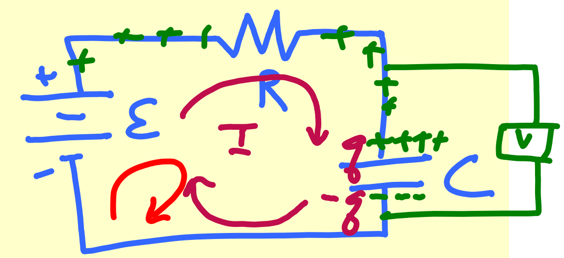
reliable overnight delivery service. (Smith went on to found Federal Express Corp.)

RC Circuits

$\Delta V = IR$

$C = \frac{Q}{\Delta V}$

- Draw a schematic of an RC circuit, and apply Kirchoff's loop rule to the circuit. Assume the capacitors starts uncharged.



$$\boxed{\varepsilon - IR - \frac{q}{C} = 0}$$

- The maximum current through the circuit is given by

$$I_{max} = \frac{\varepsilon}{R} \quad @ \quad t = 0$$

- The maximum charge on each plate is given by

$$Q_{max} = C\varepsilon \quad @ \quad t = \infty$$

Charging the Capacitor

- Write a differential equation for the circuit and solve for q(t) and I(t).

$$\left(\mathcal{E} - \frac{dq}{dt}R - \frac{q}{C} = 0\right)\frac{1}{R}$$

$$\frac{\mathcal{E}}{R} - \frac{q}{RC} = \frac{dq}{dt}$$

$$\frac{\mathcal{E}C - q}{RC} = \frac{dq}{dt}$$

$$-\int_0^{t_1}\frac{dt}{RC} = \int_0^{q_1}\frac{dq}{q-\mathcal{E}C}$$

$$-\frac{t_1}{RC} = \ln(q-\mathcal{E}C)\Big|_0^{q_1}$$

$$-\frac{t_1}{RC} = \ln(q_1-\mathcal{E}C) - \ln(-\mathcal{E}C)$$

$$-\frac{t_1}{RC} = \ln\left(\frac{q_1-\mathcal{E}C}{-\mathcal{E}C}\right)$$

$$e^{-\frac{t_1}{RC}} = \frac{q_1-\mathcal{E}C}{-\mathcal{E}C}$$

$$-\mathcal{E}Ce^{-\frac{t_1}{RC}} = q_1 - \mathcal{E}C$$

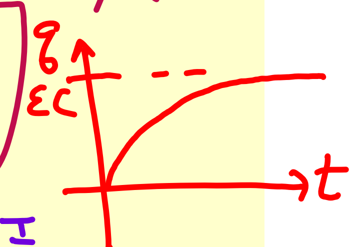
$$\boxed{q_1(t) = \mathcal{E}C\left(1-e^{-\frac{t}{RC}}\right)}$$

$$q(0) = 0$$

$$q(\infty) = Q_{MAX} = \mathcal{E}C$$

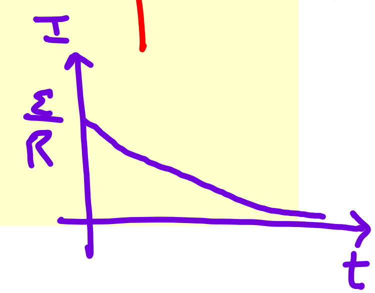
$$I(t) = \frac{dq}{dt} = \mathcal{E}\cancel{C}\left(+e^{-\frac{t}{RC}}\right)\left(\frac{1}{R\cancel{C}}\right)$$

$$\boxed{I(t) = \frac{\mathcal{E}}{R}e^{-\frac{t}{RC}}}$$



$$I(0) = I_{MAX} = \frac{\mathcal{E}}{R}$$

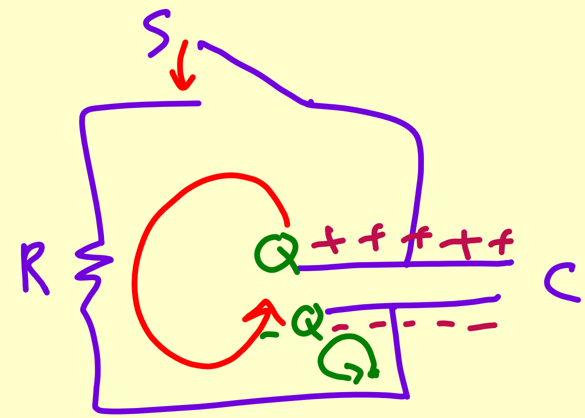
$$I(\infty) = 0$$



time constant: $\tau = RC$, time for I_0 to decrease to $\frac{1}{e} I_0$

Discharging a Capacitor

- Consider a capacitor carrying an initial charge, Q, connected to a resistor, R, and a switch. Write a differential equation for the circuit and solve for q(t) and I(t).



$$\left(\frac{q}{c} - IR = 0\right)\frac{1}{R}$$

$$\boxed{I = -\frac{dq}{dt}}$$

$$\frac{-q}{RC} = \frac{dq}{dt}$$

$$\int_0^{t_1} \frac{-dt}{RC} = \int_Q^{q_1} \frac{dq}{q}$$

$$\frac{-t_1}{RC} = \ln q \Big|_Q^{q_1}$$

$$-\frac{t_1}{RC} = \ln\left(\frac{q_1}{Q}\right)$$

$$e^{-\frac{t}{RC}} = \frac{q_1}{Q}$$

$$\boxed{Q e^{-\frac{t}{RC}} = q_1}$$

$q(0) = Q$

$q(\infty) = 0$

$$\boxed{I(t) = \frac{-dq}{dt} = \frac{Q}{RC} e^{-\frac{t}{RC}}}$$

$I(0) = \frac{Q}{RC}$

$I(\infty) = 0$

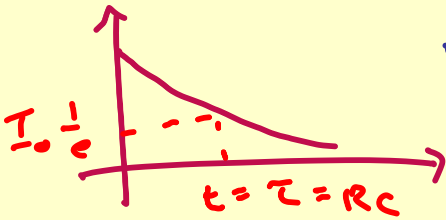
Example

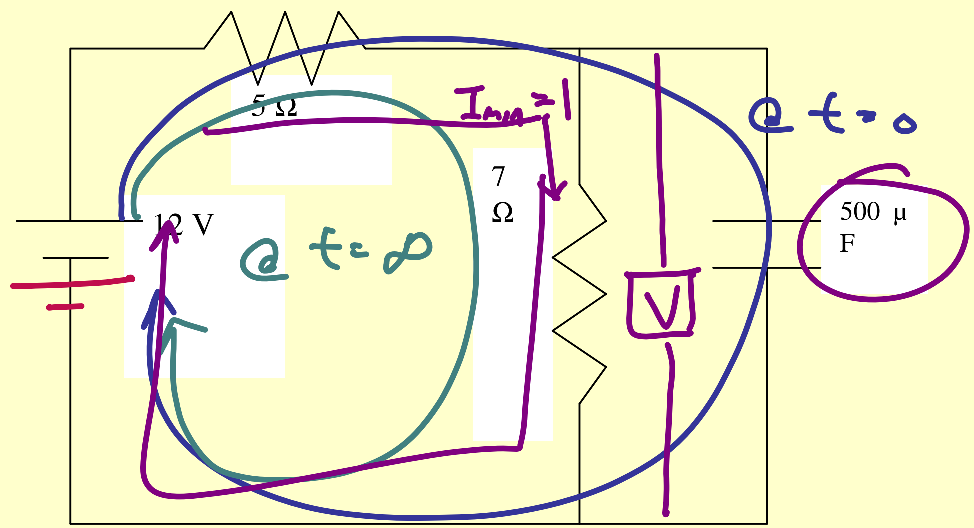
1. An uncharged capacitor is connected in a circuit as shown below.

$I = \frac{\varepsilon}{R} e^{-\frac{t}{\tau}}$ $\quad$ $\underline{t = \tau}$

$I = \frac{\varepsilon}{R} e^{-\frac{\tau}{\tau}} = \frac{\varepsilon}{R} \frac{1}{e}$

$e^{-1} = \frac{1}{e}$





a) **Find the time constant of the circuit** $\tau = RC = 5(500 \times 10^{-6}) = .0025\ s$

b) **Find the maximum current in the circuit.** @ $t = 0$ $\quad I_{MAX} = \frac{12}{5} = 2.4A$

c) **Find the minimum current in the circuit.** $I_{min} = \frac{12}{12} = 1\ A$

d) **Find the maximum charge on the capacitor.**

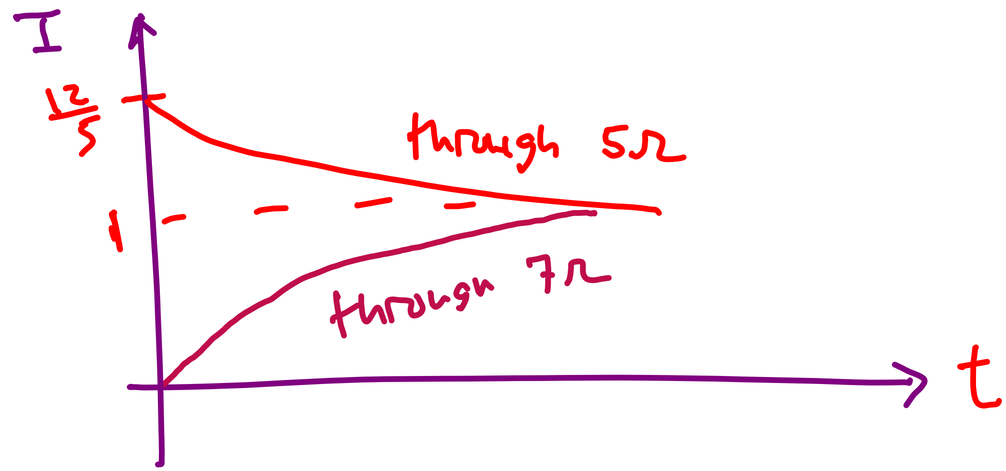
e) **Graph the current through each resistor vs. time.**

d) $C = \dfrac{Q}{\Delta V} \Rightarrow Q = C\Delta V$

$\Delta V = 1(7) = 7\ V$ across // branch

$Q = 7(500 \times 10^{-6}) = \boxed{.0035\ C}$

e)



A game C.I.