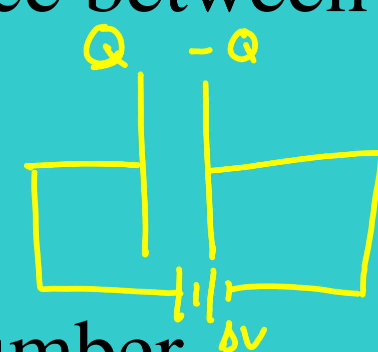
Capacitance

The real voyage of discovery consists not in seeking new landscape but having new eyes. *Proust*

Capacitance

- A capacitor stores electric charge. It consists of two conductors separated by an insulator.
- The capacitance, C, is the ratio of the magnitude of the charge on either conductor (plate) to the magnitude of the potential difference between them.

$$C \equiv \left|\frac{Q}{\Delta V}\right|$$



- Capacitance is always a positive number.
- Units are farad (F); 1 F = 1 C/V
- More commonly use μF or pF.

$\times 10^{-6}$ $\times 10^{-12}$

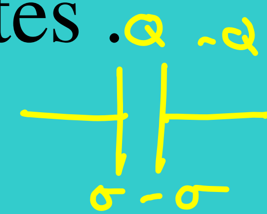
- Consider an isolated spherical conductor of radius R and charge Q. Assume second conductor is a concentric hollow sphere of infinite radius. → $V_\infty = 0$

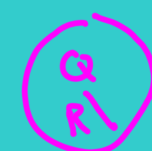
$$C = Q/\Delta V = \frac{Q}{\frac{k_e Q}{R}} = \frac{R}{k_e} = 4\pi\epsilon_0 R$$

- The capacitance of a pair of conductors depends on the geometry of a capacitor.

1. Determine C for parallel plates.

$V = \frac{kQ}{r}$





$E = \frac{\sigma}{\epsilon_0}$ for 2 plates

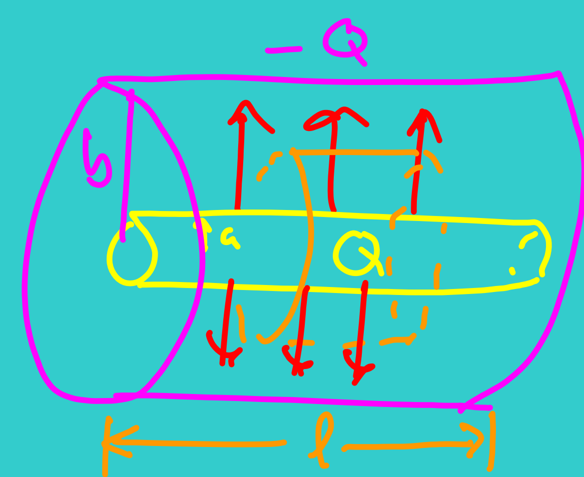
$$E = \frac{Q}{\epsilon_0 A}$$

$$\Delta V = E \cdot d = \frac{Qd}{\epsilon_0 A}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{Qd}{\epsilon_0 A}} = \frac{\epsilon_0 A}{d}$$

A - area of plates
d - distance between plates

2. A solid cylindrical conductor of radius a and charge Q is coaxial with a cylindrical shell of negligible thickness, radius, $b>a$, and charge $-Q$. Find the capacitance of this cylindrical capacitor if its length is L. ℓ



$$\oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$$

$$E(2\pi r \ell) = \frac{\lambda \ell}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k_e\lambda}{r}$$

$$C = \frac{Q}{\Delta V}$$

$$\Delta V = -\int E \cdot dr = -\int_a^b \frac{2k_e\lambda}{r}\,dr$$

$$\int \frac{dr}{r} = \ln r = \ln(b) - \ln(a) = \ln\left(\frac{b}{a}\right)$$

$$\Delta V = -2k_e\lambda \ln\left(\frac{b}{a}\right) \Rightarrow C = \left|\frac{Q}{-2k_e\lambda \ln\left(\frac{b}{a}\right)}\right| = \frac{Q}{2k_e \frac{Q}{\ell} \ln\left(\frac{b}{a}\right)}$$

$$C = \frac{\ell}{2k_e \ln\left(\frac{b}{a}\right)}$$

3. A spherical capacitor consists of a spherical conducting shell of radius b and charge $-Q$ concentric with a smaller conducting sphere of radius a and charge Q. Find the capacitance of this device.

$$E = \frac{kQ}{r^2}$$

$$\Delta V = -\int E \cdot dr = -\int_a^b \frac{kQ}{r^2}\, dr$$

$$\Delta V = +\frac{kQ}{r}\Big|_a^b = kQ\left(\frac{1}{b} - \frac{1}{a}\right)$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q\left(\frac{1}{b} - \frac{1}{a}\right)} = \boxed{\frac{ab}{k(a-b)}}$$