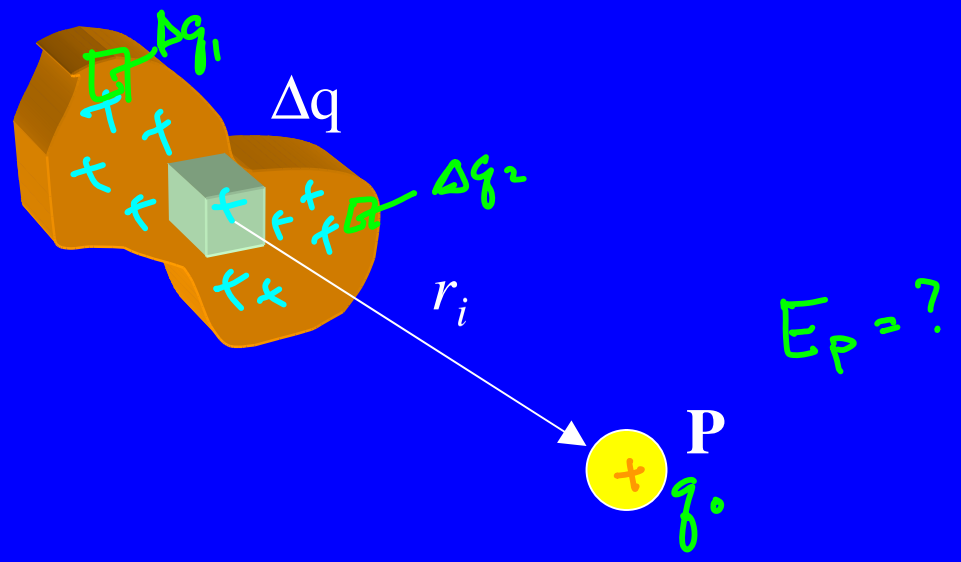
Charge Distribution

If God had meant for us to be naked,
we would have been born that way.

Electric Field of a Continuous Charge Distribution

For continuous charge distribution we

1. Divide the charge distribution into small elements that each contain Δq.
2. We use equation for E to calculate electric field due to one element at point P.
3. We then determine the total E due to all elements.



Continuous Distribution

$E = \frac{kq}{r^2}$ is field due to a point charge

- $\Delta E = \frac{k_e \Delta q}{r^2} \hat{\mathbf{r}}$ for each Δq
- $\Delta E = k_e \sum_i \frac{\Delta q_i}{r_i^2} \hat{\mathbf{r}}_i$

Because the distribution is continuous the total field at P in the limit $\Delta q_i \rightarrow 0$ is

$$E = k_e \lim_{\Delta q_i \to 0} \sum \frac{\Delta q_i}{r_i^2} \hat{\mathbf{r}}_i$$

$$\boxed{E = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}}}$$ for distributed charge

Charge Density

$\lambda = \frac{q}{\ell}$

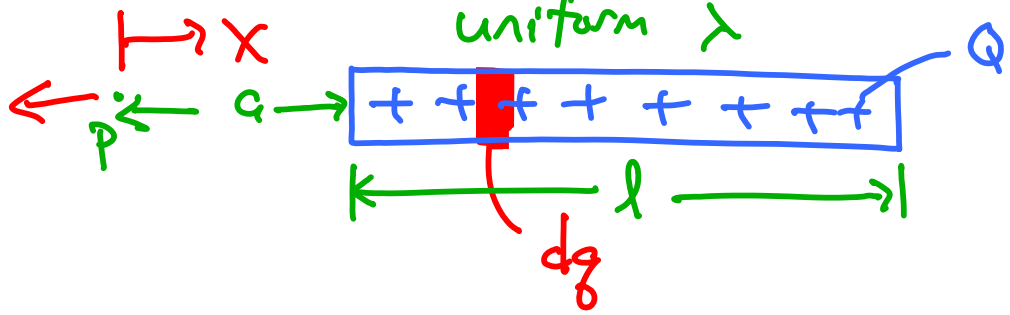
- For uniform density:
  - Volume charge density, $\rho \equiv Q/V$ (C/m^3)
  - Surface charge density, $\sigma \equiv Q/A$ (C/m^2)
  - Linear charge density, $\lambda \equiv Q/L$ (C/m)

- For nonuniform densities
  - $\rho = dQ/dV$
  - $\sigma = dQ/dA$
  - $\lambda = dQ/dL \Rightarrow \lambda dx = dq$

- Examples 23.7, 23.8 & 23.9 are great (pages 724-726)

23.7

$E_p = ?$



$$E_p = k\int \frac{dq}{x^2}$$

$$E_p = k\int_a^{a+l} \frac{\lambda\, dx}{x^2} = k\lambda \int_a^{l+a} x^{-2}dx = -k\lambda\, x^{-1}\Big|_a^{l+a}$$

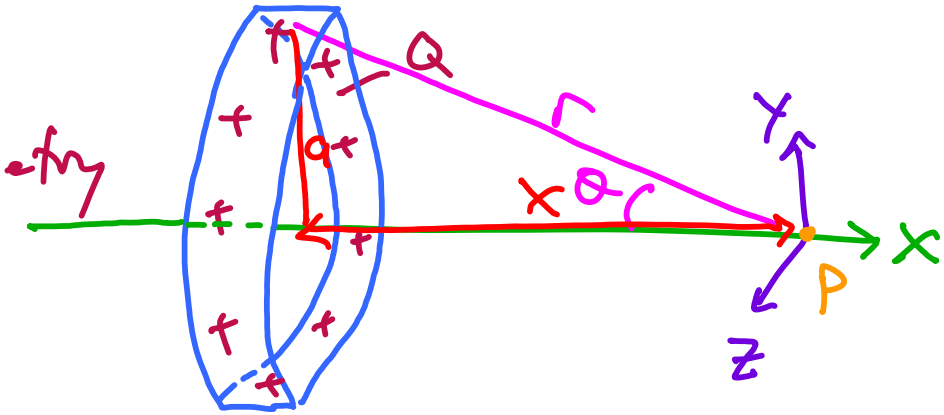
$$\boxed{E_p = -k\lambda\left(\frac{1}{l+a} - \frac{1}{a}\right) = -k\left(\frac{Q}{l}\right)\left(\frac{1}{l+a} - \frac{1}{a}\right)}$$

23.8

Q, q, x given

$E_p = ?$

$E_y = E_z = 0$ by symmetry



$E_x = E\cos\theta$

$r^2 = a^2 + x^2$

$$E_x = k\int \frac{dq}{r^2}\cos\theta = k\int \frac{dq}{(a^2+x^2)}\,\frac{x}{(a^2+x^2)^{\frac{1}{2}}}$$

$$E_x = \frac{k\,x}{(a^2+x^2)^{\frac{3}{2}}} \int_0^q dq = \boxed{\frac{k\,x\,Q}{(a^2+x^2)^{\frac{3}{2}}}}$$

23.9 $dA = 2\pi r\,dr$

$dq = \sigma\,dA$

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$$\boxed{E = \frac{\sigma}{2\epsilon_0} \text{ for sheet}}$$

$x \ll R$