Electric Potential due to Continuous Charge Distributions & due to a Charge Conductor

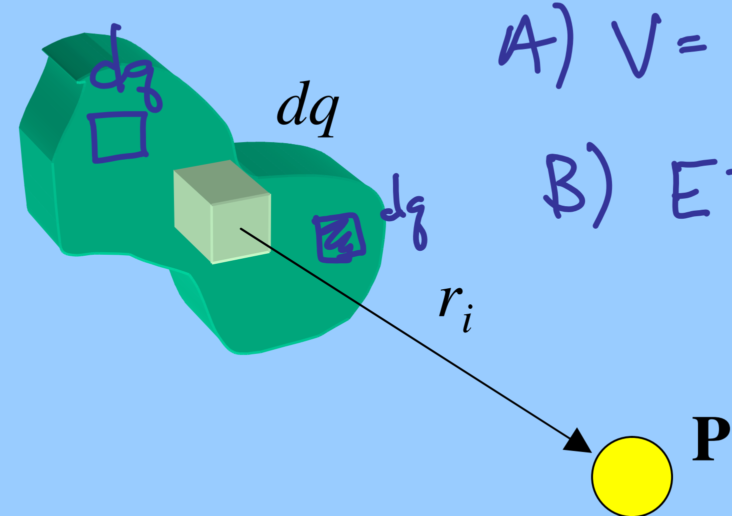
Don’t cheer, boys, those poor devils are dying. *Captain J.W. Philip*

Continuous Charge Distribution

- We can calculate the electric potential due to a continuous charge distribution two ways.
- Method #1 – consider potential due to small charge, dq

A) $V = k_e \int \frac{dq}{r}$

B) $E = -\frac{dV}{dr}$



- $\int dV = k_e \int dq/r$
- Integrate to find potential due to all elements.

$V = \frac{k_e Q}{r}$

- $\boxed{V = k_e \int dq/r}$

One more Time

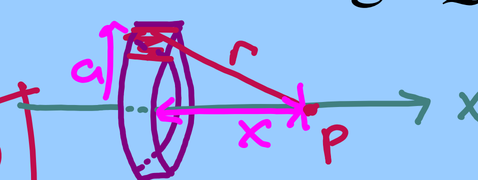
- Method #2 – If the E-field is already known due to other considerations, such as Gauss's Law, we can calculate the electric potential from A. $\oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$, get E

B. $\boxed{\Delta V = -\int \mathbf{E} \cdot d\mathbf{s}}$

1. Point P is located on the perpendicular central axis of a uniformly charged ring of radius a and total charge Q.
   a) Find electric potential at P.
   b) Find E-field at P.

a) $V = k_e \int \frac{dq}{r}$

$\boxed{V = \frac{k_e}{\sqrt{a^2+x^2}} Q}$

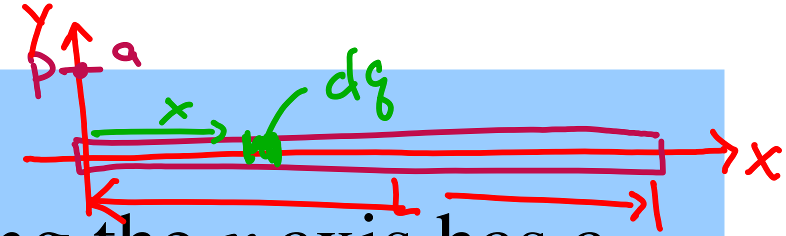


2. Consider a uniformly charged disk of radius a and total charge Q. Point P is located on the perpendicular central axis. skip
   a) Find electric potential at P.
   b) Find E-field at P.

1 b) $E = -\frac{dV}{dx}$

$E = + \frac{k_e Q}{(a^2+x^2)^{\frac{3}{2}}} (\cancel{2}x)\left(\frac{1}{\cancel{2}}\right)$

$\boxed{E = \frac{k_e Q x}{(a^2+x^2)^{\frac{3}{2}}}}$



3. A rod of length, L, located along the x axis has a total charge Q and uniform charge density, λ. Find the electric potential at point P located on the y axis at (0, a).

$$V = k_e \int \frac{dq}{r} = k_e \int_0^L \frac{\lambda\, dx}{(a^2+x^2)^{\frac{1}{2}}}$$

$$\boxed{V = k_e \lambda \ln\left[\frac{L + (L^2+a^2)^{\frac{1}{2}}}{a}\right]}$$

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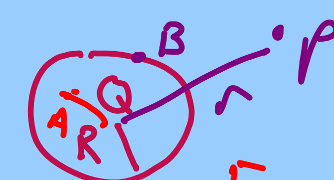
4. A insulating solid sphere of radius R has a uniform positive volume density and a total charge Q.

a) Find the electric potential at a point outside the sphere.

b) Find the electric potential at a point inside the sphere.

a) $\boxed{V_P = \frac{k_e Q}{r}} \Rightarrow \boxed{V_B = \frac{k_e Q}{R}}$



b) $V_A - V_B = \Delta V = -\int E \cdot dr = -\int_R^{r_A} \frac{k_e Q r}{R^3}\, dr$

$E = \frac{k_e Q r}{R^3}$

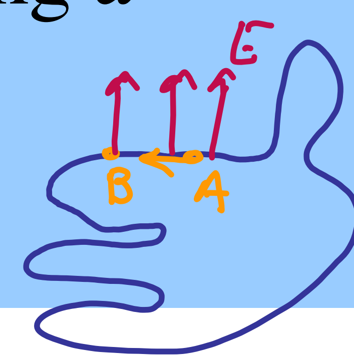
$$V_A - \frac{k_e Q}{R} = \frac{k_e Q}{R^3} \cdot \frac{1}{2} r^2 \Big|_R^{r_A} \Rightarrow \boxed{V_A = \frac{k_e Q}{2R}\left(3 - \frac{r_A^2}{R^2}\right)}$$

Potential due to Charged Conductor

- From before the E-field inside a charged conductor at equilibrium is ___0___, the E-field just outside is ___$\perp$___ to the surface, and the excess charge resides on the ___surface___.

- Every point on the surface of a charged conductor in equilibrium is at the same ___potential___.

- Consider points A and B on surface. Along a surface path E is $\perp$ to $ds \Rightarrow E \cdot ds = 0$
- $\Rightarrow V_B - V_A = -\int \mathbf{E} \cdot d\mathbf{s} = 0$

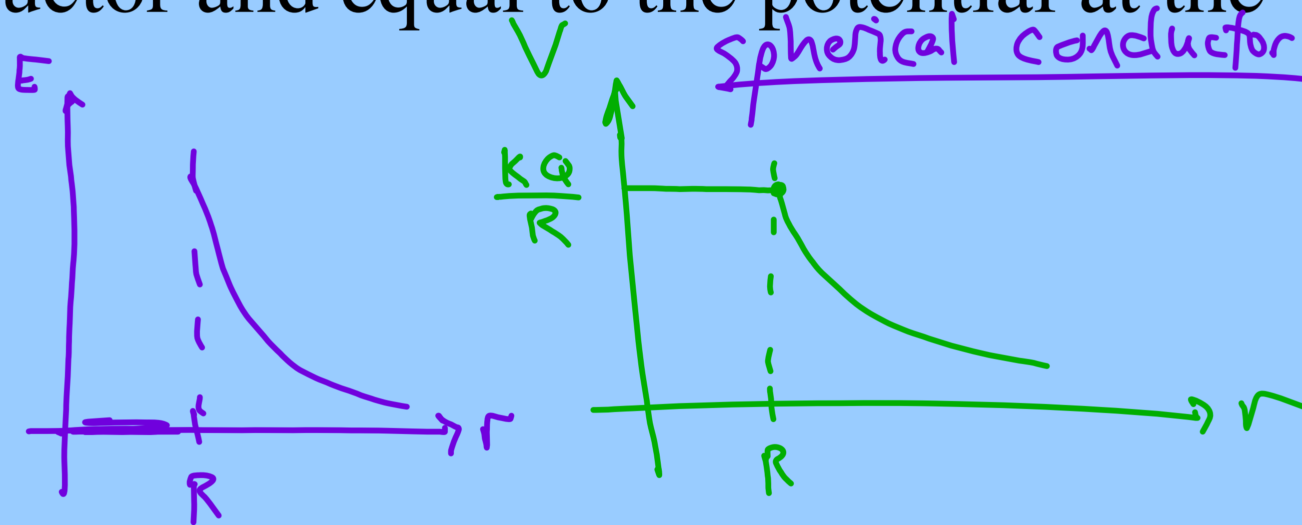


Charge Conductors

- The physical surface is an equipotential surface.
- The E-field inside is 0, therefore the relationship $\boxed{E = -dV/dr}$ implies that V is constant everywhere inside the conductor and equal to the potential at the surface.

Spherical conductor

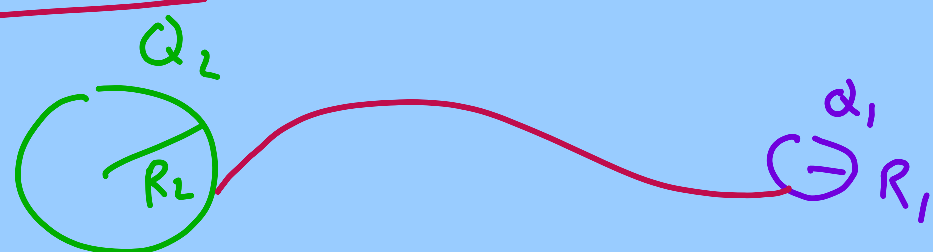
- Graph V vs. r



- The E-field and surface charge density is large near sharp points of a charge conductor.

Example 5

Two spherical conductors of radii R_1 & R_2 are separated by a distance much greater than the radius of either sphere. The spheres are connected by a conducting wire. The charges on the spheres in equilibrium are Q_1 and Q_2 respectively and they are uniformly charged. Find the ratio of the magnitudes of the E-fields at the surfaces of the spheres.



$$\boxed{V_1 = V_2}$$

$$\frac{\cancel{k_e} Q_1}{R_1} = \frac{\cancel{k_e} Q_2}{R_2}$$

$$\frac{Q_1}{Q_2} = \frac{R_1}{R_2}$$

$$\frac{E_1}{E_2} = \frac{\cancel{k_e}\frac{Q_1}{R_1^2}}{\cancel{k_e}\frac{Q_2}{R_2^2}} = \left(\frac{Q_1}{Q_2}\right)\cdot\frac{R_2^2}{R_1^2} = \frac{\cancel{R_1}}{\cancel{R_2}}\cdot\frac{R_2^{\cancel{2}}}{R_1^{\cancel{2}}}$$

$$\boxed{\frac{E_1}{E_2} = \frac{R_2}{R_1}}$$