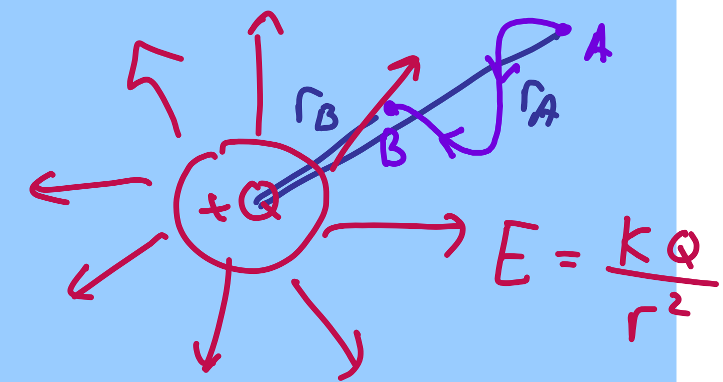
Potential Due to Point Charges, and E $\Leftrightarrow$ V

This is a place where most people still understand the crucial secret of human happiness: that it's better to do a few things slowly, than a lot of things fast. *McCarthy*

Potential due to a point charge

- A charge moves from A to B near a point charge.
- Solve for ΔV.



$$\Delta V = -\int_{r_A}^{r_B} E \cdot dr = -\int_{r_A}^{r_B} \frac{KQ}{r^2}\, dr$$

$$\Delta V = KQ\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

assume A is $\infty \Rightarrow V_A = 0$

$$\boxed{V = \frac{KQ}{r}} \qquad V_{TOT} = \sum_i \frac{KQ_i}{r_i}$$

- The potential energy of a multi-particle system is

$$V = \frac{U}{q_0} \Rightarrow U = Vq$$

$$U = k_e\left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}}\right]$$

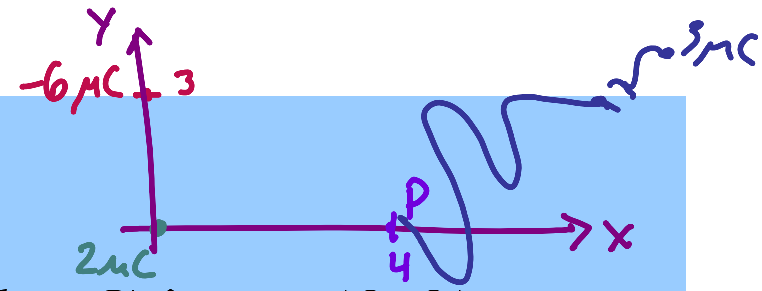
- Main Hint: start @ ∞ work inward

E-Field from Potential

- If E-field only has an x-component then $\boxed{E_x = -\frac{dV}{dx}}$
- $\boxed{dV = -E \cdot dx}$
- For radial symmetry, $E_r = -\frac{dV}{dr}$
- Equipotential lines are $\underline{\perp}$ to E-field lines.
  - If test charge moves along an equipotential line $\Delta V = 0$
    no work was done.
- $E_x = -\frac{\partial V}{\partial x}$ $E_y = -\frac{\partial V}{\partial y}$ $E_z = -\frac{\partial V}{\partial z}$

∂ - partial derivative

Examples



1. A 2 μC is at the origin, and a –6 μC is at (0,3).
   a) Find the potential at P, (4,0).
   b) Find ΔU for a 3 μC charge from infinity to P.

a) $V = \frac{k(2\times10^{-6})}{4} + \frac{k(-6\times10^{-6})}{5}$

$\boxed{V = -6.29\times10^{3}\ V}$

b) $\Delta V = \frac{\Delta U}{q_0}$

$\Delta U = \Delta V q_0$

$\Delta U = (-6290)(3\times10^{-6})$

$\boxed{\Delta U = -18.9\times10^{-3}\ J}$

2. V = 3x²y + y² + yz. Find E.

$$E = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

$$E = -(\underbrace{3y\cdot 2x}_{6xy})\hat{i} - (3x^2 + 2y + z)\hat{j} - (y)\hat{k}$$