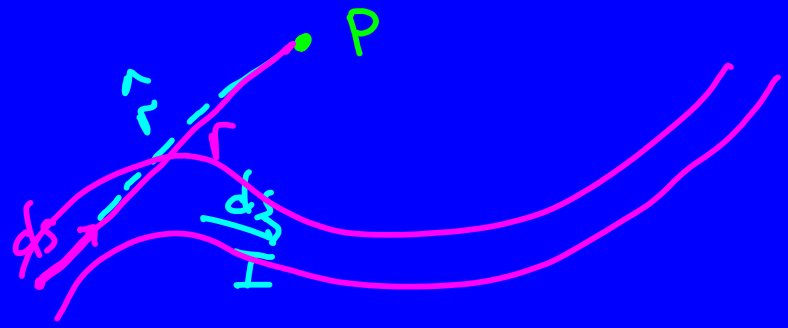
Biot-Savart

The woods would be very silent if no bird sang there except those who sing the best. *Thoreau*.

"Stocks have reached what looks like a permanently high plateau." Irving Fisher, Professor of Economics, Yale University, 1929.

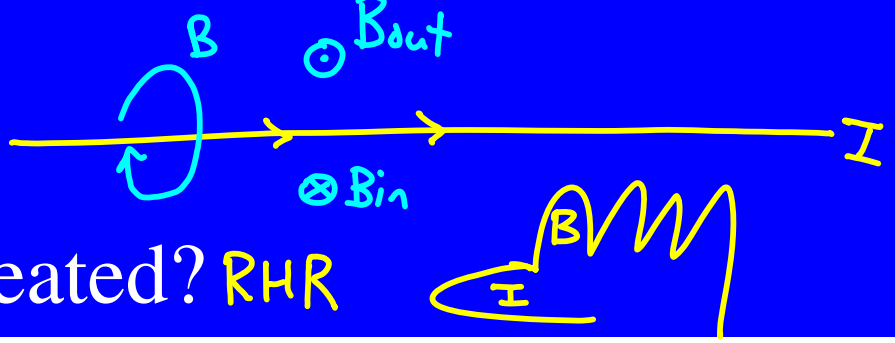
Biot-Savart

- Moving charge deflects a compass needle. Biot and Savart performed experiments to find the value of the magnetic field due to a current.



- $dB = \dfrac{\mu_o I\, d\vec{s} \times \hat{r}}{4\pi r^2}$
- $\mu_o = 4\pi \text{ X } 10^{-7}$ T·m/A, permeability of free space
- How do we find total B? integrate
- ε_o permittivity of free space

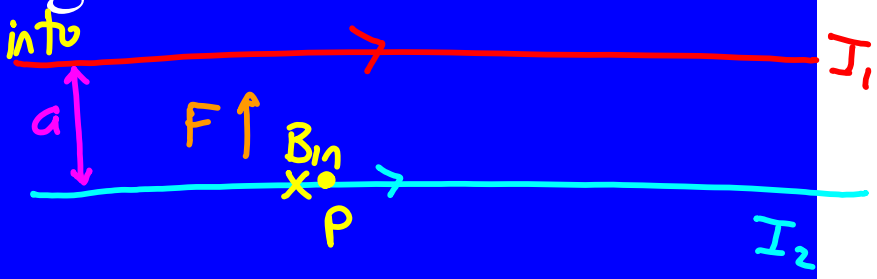
$$B = \frac{\mu_o I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$



- Direction of the B-field created? RHR

Parallel Conductors

- Imagine two long conductors carrying currents. Calculate the force per unit length between the conductors. $B_P = \frac{\mu_0 I_1}{2\pi a}$ (ex #1)



$F = I \ell \times B$

$\frac{F}{\ell} = I_2 \left(\frac{\mu_0 I_1}{2\pi a}\right) = \frac{\mu_0 I_1 I_2}{2\pi a}$

- Define Ampere: When the force/length is 2 X 10^{-7} N/m, and the wires are 1 m apart, then the current in each wire is defined as 1 A.

- Define Coulomb: When a steady 1 A current flows the quantity of charge flow through the cross-section in 1 s is 1 C.

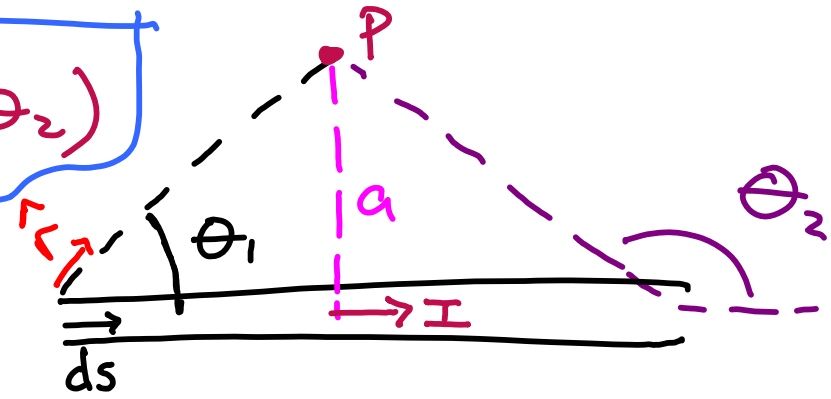
Examples

1. Determine B at P for a wire carrying I.

2. Calculate B at O for wire shown.

3. Consider a circular loop in yz plane and carrying I. Calculate B at P.

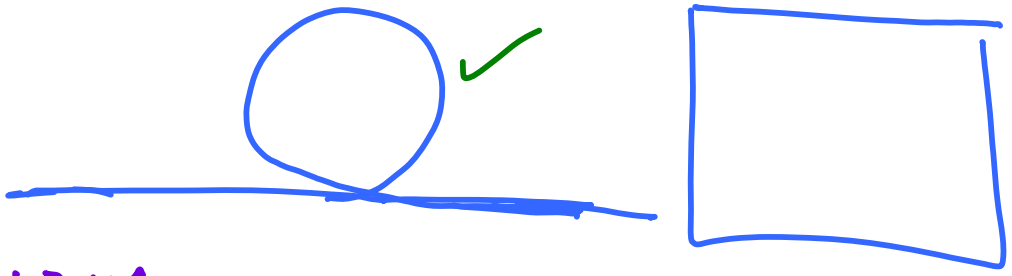
1. $B_P = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2)$



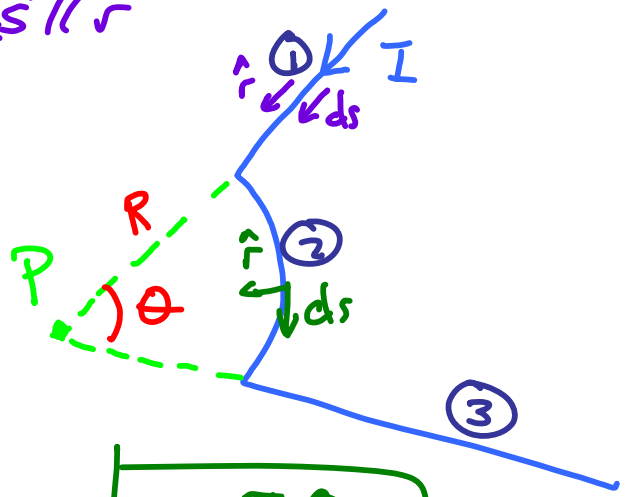
finite length of wire

infinite wire

$B_P = \frac{\mu_0 I}{2\pi a}$



2. $B_{1\&3} = 0$ b/c $d\vec{s} \parallel \hat{r}$



$B_2 = \frac{\mu_0 I}{4\pi}\int \frac{ds}{R^2}$

$B_2 = \frac{\mu_0 I}{4\pi R^2} s$

$B_2 = \frac{\mu_0 I}{4\pi R^2} R\theta = \boxed{\frac{\mu_0 I \theta}{4\pi R}}$

3. $r = \sqrt{R^2 + x^2}$ for each $d\vec{s}$

$$B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{\frac{3}{2}}}$$