Faraday’s & Motional emf

If you cannot explain what you did, then the doing has not been worth it. *Heisenberg*

Faraday's Law

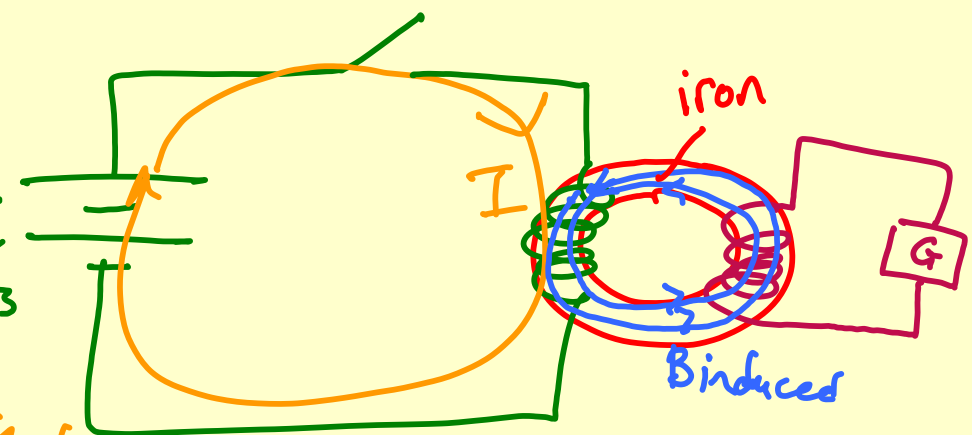
- We have shown how magnetic field are produced by moving charge. We will now explore how changes in B-fields produce E-fields (emf, currents, etc.)

- We can induce a current without a battery. DEMO.

- Faraday's experiment. $\mathcal{E}$

close switch: needle deflects
& returns to zero

open switch: needle deflects
other direction & returns to zero.



Faraday's Law II

ΔV

- Faraday's Law of Induction: the *emf* induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit.

$$\mathcal{E} = -N \frac{d\Phi_B}{dt} = -N \frac{d}{dt}(B \cdot A)$$

N – # of coils

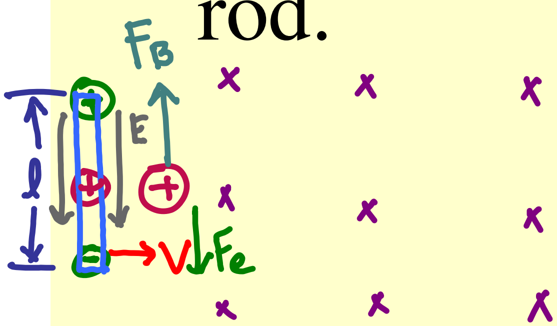
- List three ways to induce a current in a wire loop that is in a B-field.

1. ΔB
2. ΔA
3. $\Delta\theta$ (orientation)

Motional emf

- Motional emf is the emf induced in a conductor moving in a constant B-field.

1. Imagine a metal bar of length ℓ moving through a uniform B-field. Solve for induced ΔV across the rod.



@ equilibrium $F_B = F_e$

$$qvB = q E$$

$$vB = \frac{\Delta V}{\ell}$$

$$\boxed{\Delta V = \mathcal{E} = B\ell v}$$

Aside

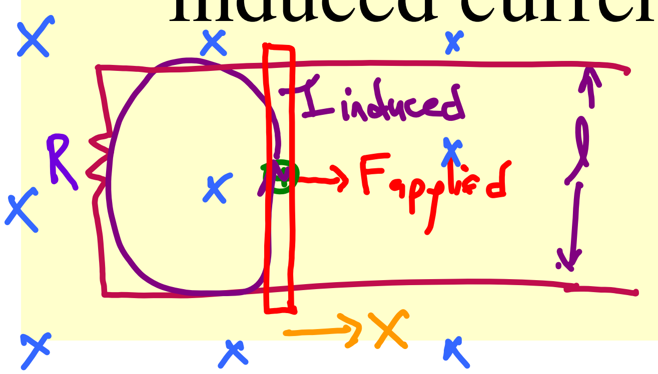
$$\Delta V = -\int \vec{E}\cdot d\vec{s}$$

$$\Delta V = -\vec{E}\cdot\vec{\ell}$$

2. Now imagine a bar on rails as shown. Find the induced current.

$$\mathcal{E} = \frac{d(B\cdot A)}{dt} = B\ell\frac{dx}{dt} = B\ell v$$

$$I = \frac{\mathcal{E}}{R} = \boxed{\frac{B\ell v}{R}}$$



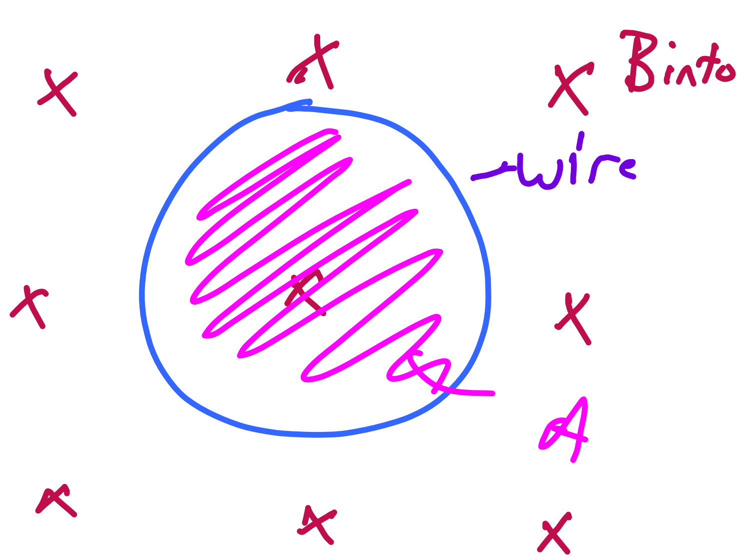
Examples

1. A loop of wire enclosed an area A is placed in a region where the B-field is perpendicular to the plane of the loop. $B = B_o e^{-at}$. Find the induced emf in the loop.
2. Copy diagram. A decreasing magnetic field induces an emf in the loop that causes the light bulbs to light. What happens to the brightness of the bulbs when the switch is closed?
3. A bar of mass *m* and length ℓ moves on two frictionless rails as shown. It is given an initial velocity v_o to the right. Find the velocity as a function of time.

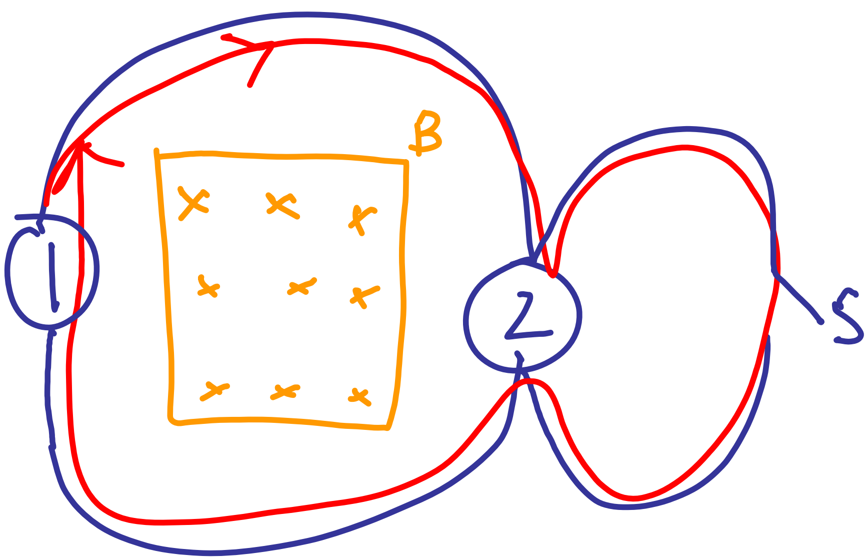
1. $B = B_0 e^{-at}$ given

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -A \frac{dB}{dt} = \boxed{A a B_0 e^{-at}}$$



2.



$\frac{dB}{dt} \neq 0$

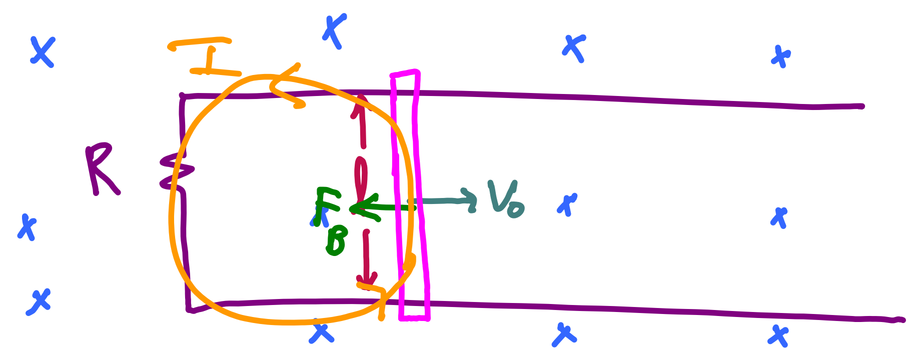
open switch: both ① & ② are on

close S while $\frac{dB}{dt} \neq 0$

① is brighter & ② is off

3. $\mathcal{E} = Blv$

$$I = \frac{\mathcal{E}}{R} = \frac{Blv}{R}$$



$$F = I\ell B = \left(\frac{B\ell v}{R}\right)\ell B$$

$$F = -\frac{B^2\ell^2 v}{R} = ma$$

$$\left(-\frac{B^2\ell^2}{mR}\right) v = \frac{dv}{dt}$$

$$-\frac{B^2\ell^2}{mR}\int_0^{t_1} dt = \int_{v_0}^{v_1} \frac{dv}{v}$$

$$-\frac{B^2\ell^2}{mR} t_1 = \ln\left(\frac{v_1}{v_0}\right)$$

$$v_1 = v_0 e^{-\frac{B^2\ell^2}{mR} t}$$