Energy, Mutual Inductance & Oscillations in LC Circuits

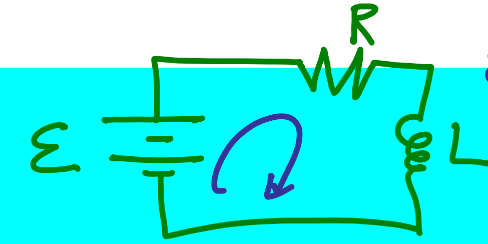
"640k ought to be enough for anybody." -- Bill Gates, 1981

If you cannot explain what you did, then the doing has not been worth it. *Heisenberg*

Time is a great teacher, but unfortunately it kills all its pupils. *Berlioz*

Not everything that counts can be counted, and not everything that can be counted counts. (Sign hanging in Einstein's office at Princeton)

Energy



$\varepsilon_L = -L\frac{dI}{dt}$

- Draw a LR circuit with a battery and write the differential equation for the loop.

$$\left(\varepsilon - IR - L\frac{dI}{dt} = 0\right) I$$

- Multiply the equation by I to get

$$I\varepsilon = I^2R + LI\frac{dI}{dt}$$

$$P_{battery} = P_{resistor} + P_{inductor}$$

$$P_L = \int \frac{dU_L}{\cancel{dt}} = \int L I \frac{dI}{\cancel{dt}}$$

$$\boxed{U_L = \frac{1}{2} L I^2}$$

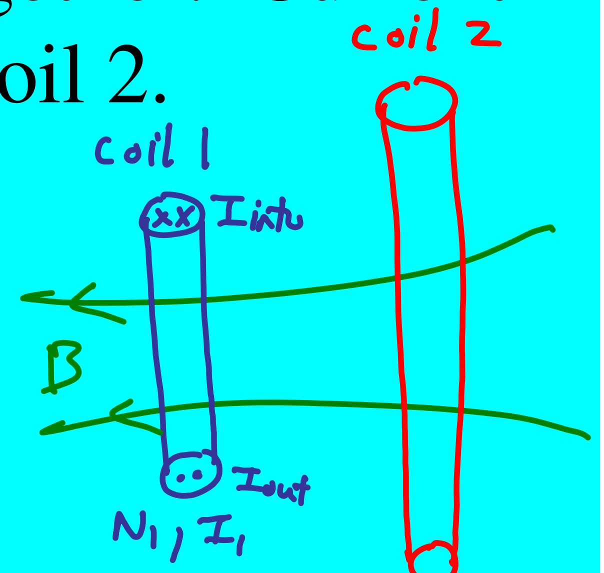
- This equation is an example of

conservation of energy

- Solve for U_L.

Mutual Inductance

- Consider two coils that are close together. Current in coil 1 will create a flux through coil 2.



- We define mutual inductance as

$$M_{12} = \frac{N_2 \Phi_{12}}{I_1}$$ of coil 2 w.r.t. coil 1

Φ_{12} - flux through 2 due to 1

- Solve for the *emf* induced in coil 2

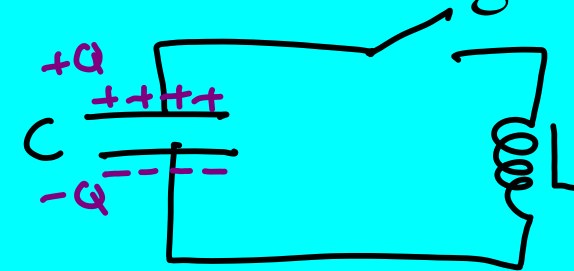
$$\mathcal{E}_2 = -N_2 \frac{d\Phi_B}{dt} = -\cancel{N_2} \frac{d}{dt}\left(\frac{M_{12} I_1}{\cancel{N_2}}\right)$$

$$\mathcal{E}_2 = M_{12} \frac{dI_1}{dt}, \quad \mathcal{E}_1 = -M_{21} \frac{dI_2}{dt}$$

- In mutual inductance, the emf induced in one coil is always proportional to the rate at which the current in the other coil is changing.

Oscillations in LC Circuits

- Draw an LC circuit with a switch and a charged capacitor.



- We will assume no energy lost by internal energy (which implies ___R___ is zero) or radiation.
- Use transparency to discuss energy transfer.
- Start with the initial energy and solve for ω.

$U_{tot} = U_C + U_L$

$U = \frac{Q^2}{2C} + \frac{LI^2}{2}$

$\frac{dU}{dt} = \frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt} = 0$

$\frac{Q}{C}\cancel{I} + L\cancel{I}\frac{dI}{dt} = 0$

$L\frac{dI}{dt} = -\frac{Q}{C}$

$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q$

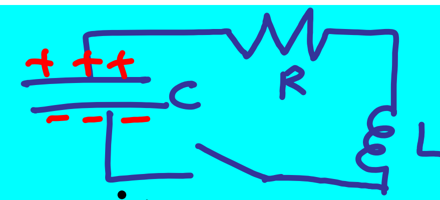
$\frac{d^2x}{dt^2} = -\omega^2 x$

$\boxed{\omega^2 = \frac{1}{LC}}$

$q(t) = Q_{max}\cos\omega t$

$I(t) = -Q\omega \sin \omega t$

RLC Oscillations



- Draw an RLC circuit with a charge capacitor.
- Use Kirchhoff's Rules to derive a differential equation.

$$L\frac{dI}{dt} + IR + \frac{Q}{C} = 0$$

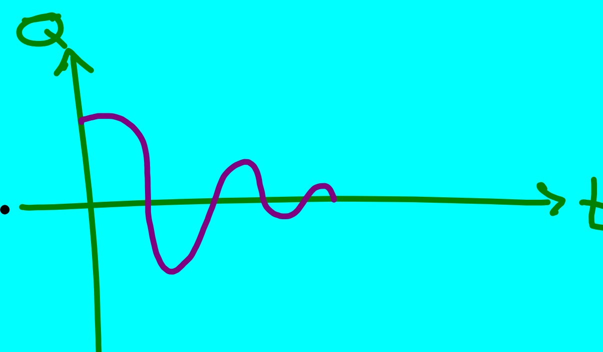
$$L\frac{d^2Q}{dt^2} + \frac{dQ}{dt}R + \frac{Q}{C} = 0$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$L \rightarrow$ inertia

$R \rightarrow$ dampening

- Compare this to a spring/mass system with a resistive force due to dampening. $C \rightarrow$ restoring force
- Draw the graph of the oscillation.



- Why doesn't capacitor just discharge until neutral?