RL Circuits

A mathematician is a machine for turning coffee into theorems. *Erdos*

Inductors

coils (solenoid) with a large self-inductance

- What is an inductor?
- An inductor always <u>opposes</u> the changes in the <u>current</u> through the circuit.
- Draw an RL circuit with a switch.

$\mathcal{E}_L = -L\frac{dI}{dt}$

- Use Kirchhoff's voltage law to get a differential equation.

$$\mathcal{E} - L\frac{dI}{dt} - IR = 0$$

$$\frac{dI}{dt} > 0 \Rightarrow \mathcal{E}_L < 0$$

Solution

- Derive the solution to the RL circuit.

$$\mathcal{E} - IR = L\frac{dI}{dt}$$

$$\int_0^{t_1} dt = L\int_0^{I_1} \frac{dI}{\mathcal{E} - IR}$$

$$\int \frac{dI}{-RI} = \frac{-1}{R}\ln(-RI)$$

$$t_1 = -\frac{L}{R}\ln(\mathcal{E} - IR)\Big|_0^{I_1}$$

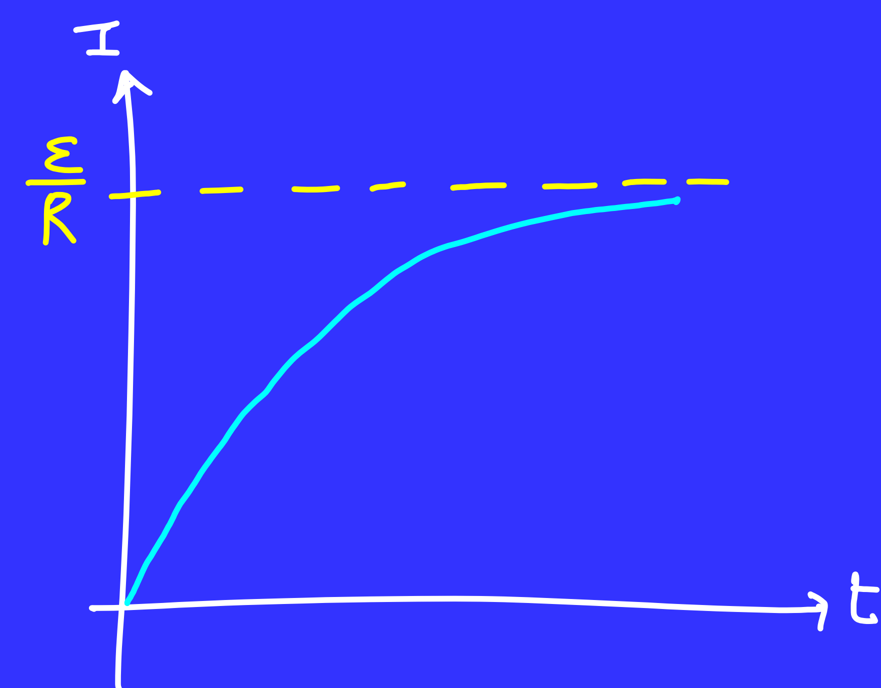
$$-\frac{Rt_1}{L} = \ln\left(\frac{\mathcal{E} - IR}{\mathcal{E}}\right)$$

$$e^{-\frac{Rt_1}{L}} = \frac{\mathcal{E} - IR}{\mathcal{E}}$$

$$\boxed{\frac{\mathcal{E}}{R}\left(1 - e^{-\frac{Rt}{L}}\right) = I}$$

$$e^{-\frac{t}{\tau}}$$

$\tau = \frac{L}{R}$ for LR circuits



Example

- Copy the circuit diagram, and describe changes in the circuit when the switches are opened/closed.

close S_1: I increases to $\frac{\varepsilon}{R}$ as shown before

close S_2 & open S_1 simultaneously:

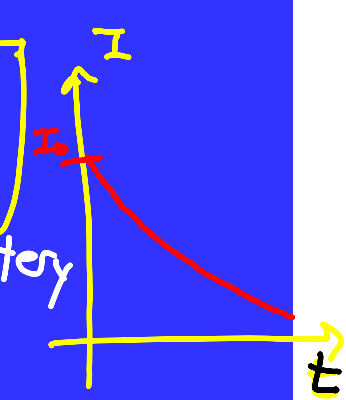
A: $IR + L\frac{dI}{dt} = 0$

$\frac{dI}{dt} < 0 \Rightarrow \varepsilon_L > 0$

$L\frac{dI}{dt} = -IR \Rightarrow \int_{I_0}^{I_1} \frac{dI}{I} = -\frac{R}{L}\int_0^{t_1} dt$

$\ln\left(\frac{I_1}{I_0}\right) = -\frac{R}{L}t_1$

$$I_1 = I_0 e^{-\frac{R}{L}t_1}$$



- $\Delta V_L = -L\frac{dI}{dt}$
- At $t = 0$, an inductor acts like an opposing battery
- A $t = \infty$, and inductor acts like a wire.

Examples

1. Two circuits similar to the previous example have different inductors. Switch 1 is closed at $t = 0$ and then switch 2 is closed. Copy the graphs shown. Which circuit has the greater inductor?

2. A 12 V battery is in series with a 30 mH inductor and a 6 ohm resistor.
   a) How long does it take the current to reach $2(1-e^{-1})$ amps?
   b) Calculate the current at 2 ms.
   c) Sketch the voltage across the inductor and the resistor.

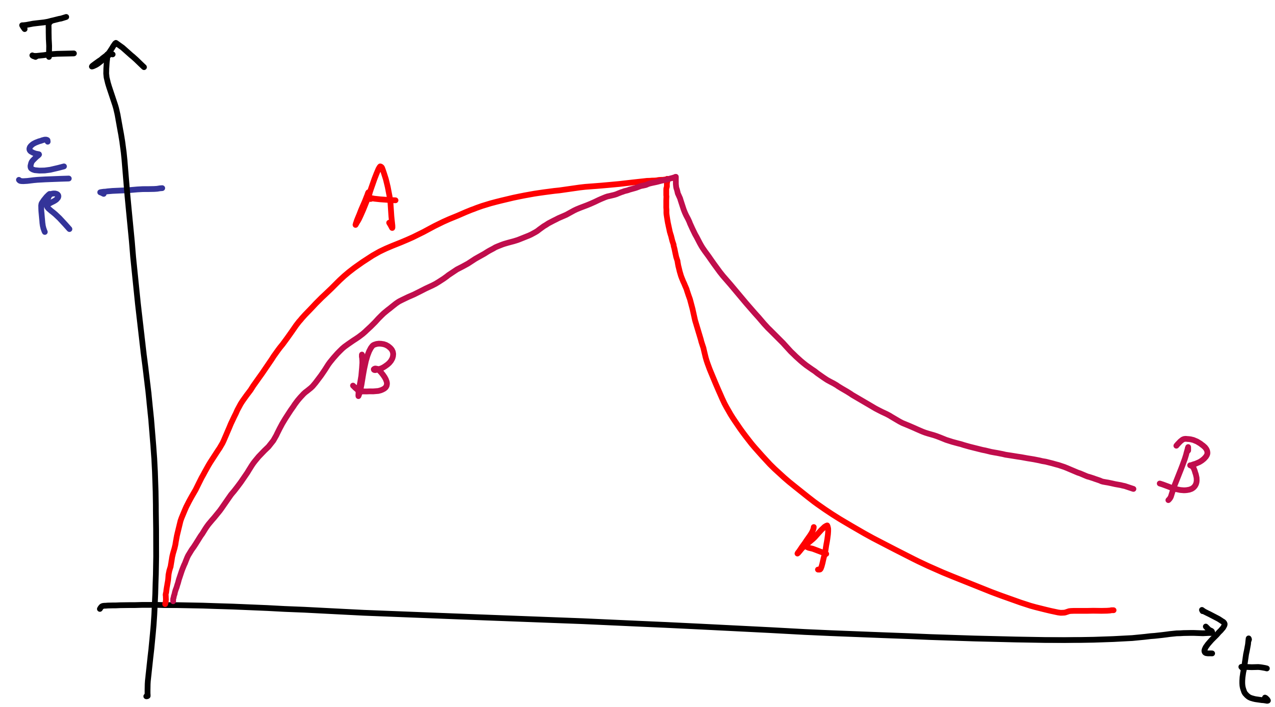
1. R is same

$L_B \overset{?}{>} L_A$

$\tau_B > \tau_A$

$\tau = \frac{L}{R}$

$L_B > L_A$

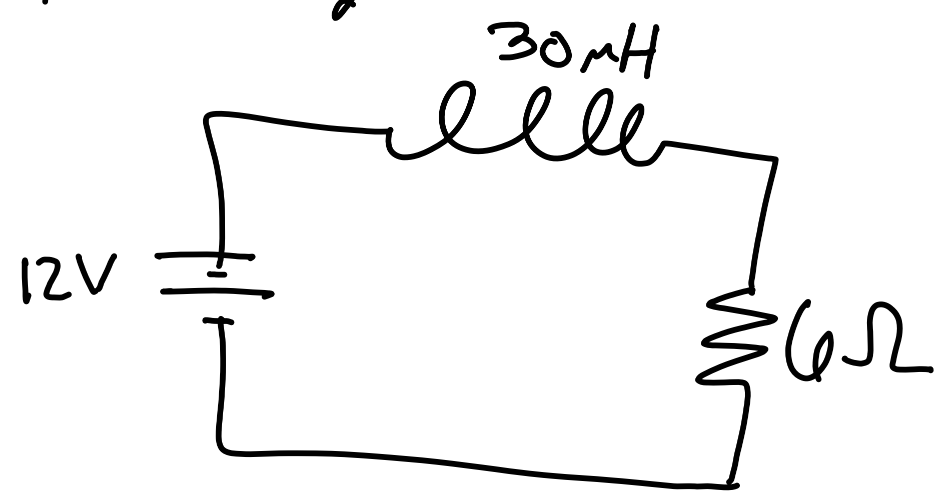


2. (a) $I = 2(1-e^{-1})$ takes how long?

$I = \frac{\varepsilon}{R}(1-e^{-\frac{t}{\tau}})$

$I = 2(1-e^{-\frac{t}{\tau}})$

$\tau = \frac{L}{R} = \frac{30}{6} = 5\ ms$



(b) $I = 2(1-e^{-\frac{2}{5}}) = .66A$

(c)