Gauss's Law!

- Maxwell's Equations

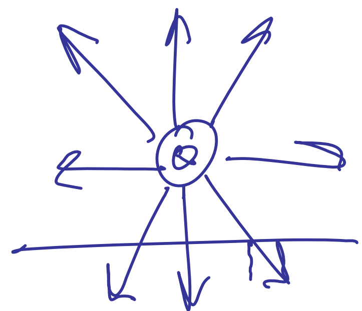
Gauss's Law
$$\oint\!\!\oint \vec{E}\cdot d\vec{S} = \frac{Q_{enc}}{\epsilon_0}$$

Faraday's Law
$$\oint \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\iint \vec{B}\cdot d\vec{S}$$

Gauss's Law of Magnetism
$$\oint\!\!\oint \vec{B}\cdot d\vec{S} = 0$$

Ampère's Law
$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d}{dt}\iint \vec{E}\cdot d\vec{S}$$

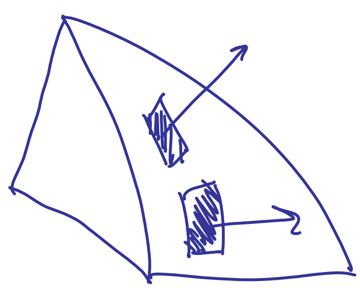
- Flux



$$\Phi = \vec{E}\cdot\vec{A}$$

units: $[N \cdot m^2 \cdot C^{-1}]$

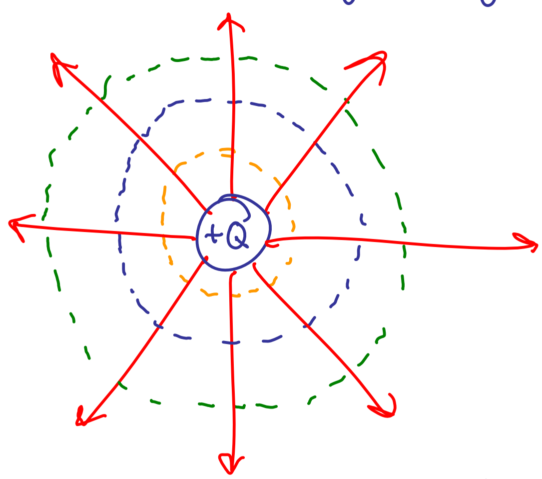
- Surface Normal/Area Vector convention:



- So - a little flux is $d\Phi_E = \vec{E}\cdot d\vec{A}$

Signs of flux: (+) (−) (0)

What kind of surface?



Why is Φ independent of distance from charge?

$E \propto \frac{1}{r^2}$

$\oiint$

$S \propto r^2$

$E \cdot S \propto \frac{1}{r^2} r^2 = 1$

Flux through dotted surface is (same)

Generalization is Gauss's Law: $\oiint \vec{E} \cdot d\vec{S} = \frac{Q_{enc}}{\epsilon_0}$

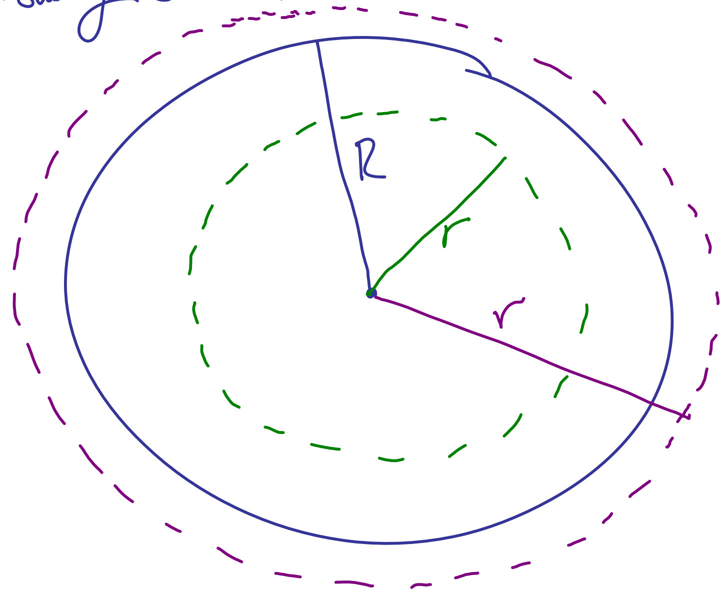
ϵ_0 is a fundamental constant: $8.85 \times 10^{-12}\ C^2 \cdot N^{-1} \cdot m^{-2}$
(permittivity of free space)

Gauss's Law is useful when one or more of the following apply:

- Symmetry
- $\vec{E} \parallel d\vec{S}$
- $\vec{E} \perp d\vec{S}$
- $\vec{E} = \vec{0}$ over S

Example ①

insulating solid sphere - uniform ρ, total charge Q



What's the E-field at some $r > R$?

$$\oint\!\!\oint_{purple} \vec{E}\cdot d\vec{S} = \frac{Q}{\epsilon_0}$$

$$\vec{E}\cdot\oint\!\!\oint d\vec{S} = \frac{Q}{\epsilon_0}$$

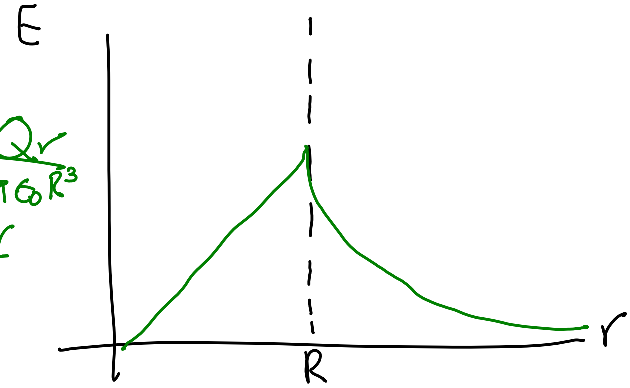
$$|\vec{E}|\,(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{kQ}{r^2}$$

What's the E-field at some $r < R$?

$$\oint\!\!\oint_{green} \vec{E}\cdot d\vec{S} = \frac{Q}{\epsilon_0}\left(\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}\right)$$

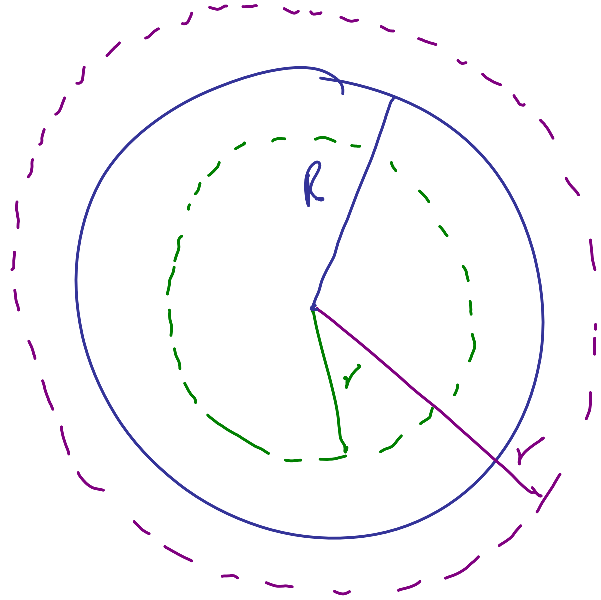
$$|E|\,4\pi r^2 = \frac{Q r^3}{\epsilon_0 R^3} \rightarrow E = \frac{Q r^3}{4\pi\epsilon_0 R^3 r^2} = \frac{Qr}{4\pi\epsilon_0 R^3} = \frac{kQr}{R^3}$$



Example ②

Conducting thin spherical shell

Q uniform over surface



What's the E-field inside the shell?

(0)

What's the E-field at some $r > R$?

$$\oiint \vec{E}\cdot d\vec{S} = \frac{Q}{\epsilon_0}$$

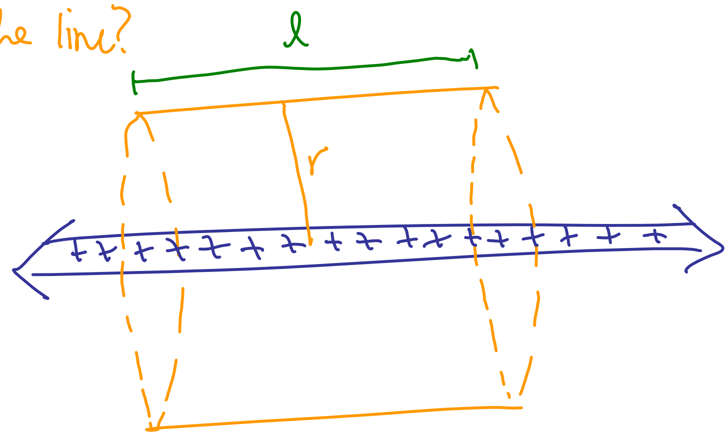
$$E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{kQ}{r^2}$$

Example ③

Infinite line of charge with λ

What's the E-field at some r from the line?



$$\oiint \vec{E}\cdot d\vec{S} = \frac{\lambda l}{\epsilon_0}$$

$$E(2\pi r \cancel{l}) = \frac{\lambda \cancel{l}}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k\lambda}{r}$$

One Last Example: ④

What's the E-field at some r away from the plane?

$Q_{enc} = \sigma A$

$\oint\!\!\oint \vec{E} \cdot d\vec{S} = \frac{\sigma A}{\epsilon_0}$

$E(2A) = \frac{\sigma A}{\epsilon_0}$

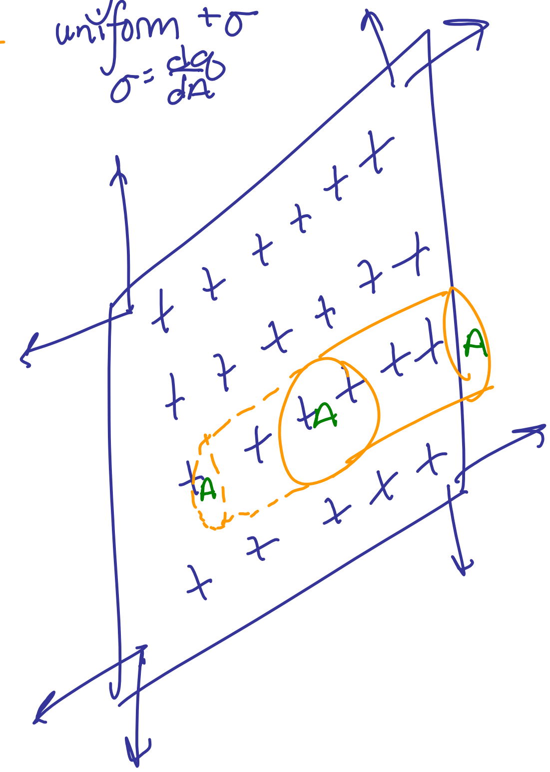
$E = \frac{\sigma}{2\epsilon_0}$

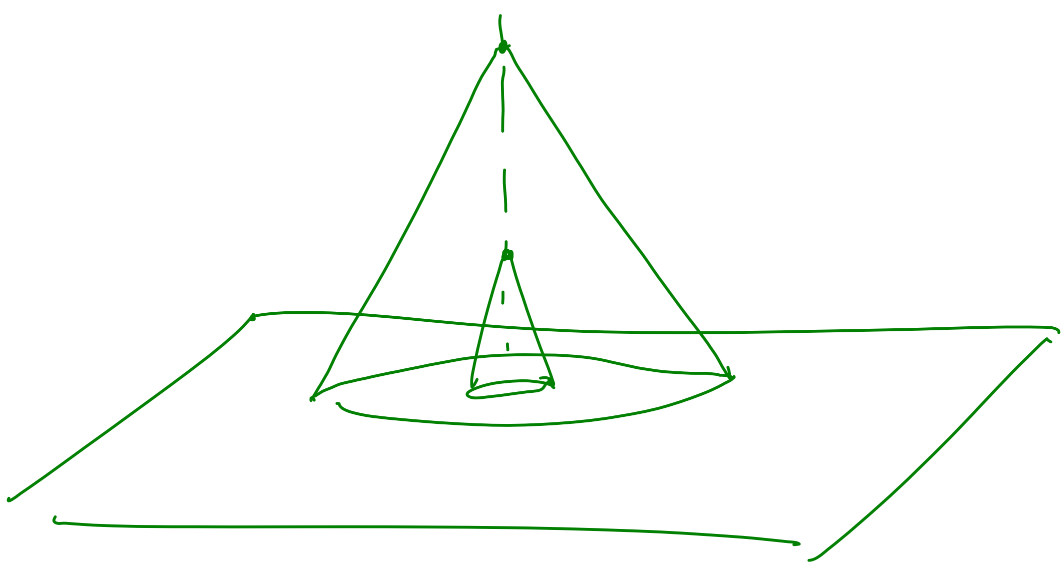
Why is there no $\boxed{\sqrt{}}$?

Infinite conducting Plane

uniform $+\sigma$

$\sigma = \frac{dq}{dA}$





Enrichment: Gauss's Law in Differential form

$$\oiint_{\partial V} \vec{E} \cdot d\vec{S} = \frac{Q_{enc}}{\epsilon_0}$$

$$Q_{enc} = \iiint_V \frac{\rho}{\epsilon_0}\, dV$$

$$= \iiint_V \nabla \cdot \vec{E}\, dV = \iiint_V \frac{\rho}{\epsilon_0}\, dV$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$