VIII Rotation

(A) $s = R\theta$, $v = R\omega$, $a_{tan} = R\alpha$

(B) θ, $\omega = \frac{d\theta}{dt}$, $\alpha = \frac{d\omega}{dt}$

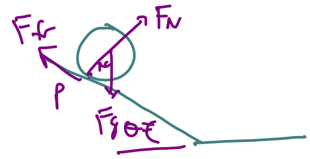
(C) constant α eq.

$$\omega_2 = \omega_1 + \alpha t$$
$$\omega_2^2 = \omega_1^2 + 2\alpha\theta$$
$$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$$

(D) Rotational Inertia

1. Point masses, $I = \sum_i m_i r_i^2$
2. Distributed mass, $I = \int r^2 dM$
   (a) $\lambda = \frac{dM}{dx} = \frac{M}{l}$
   (b) $\sigma = \frac{dM}{dA}$
3. $I = I_{C.M.} + Md^2$, // axis theorem

(E) $\tau = r \times F = I\alpha$



(F) $K_{rot} = \frac{1}{2} I\omega^2$

(G) Rolling

1. $K_{TOT} = K_r + K_t = \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} m V_{cm}^2$
2. Energy: $E_1 = E_2$
3. $\sum \tau_p = I_p \alpha$ to get acc.
4. $\sum F = ma$ to get μ.

ex) Cylinder of M, R, get acc. & μ, on a slope, θ.

(a) $\sum \tau_p = I_p \alpha$

$I_{cylinder}$

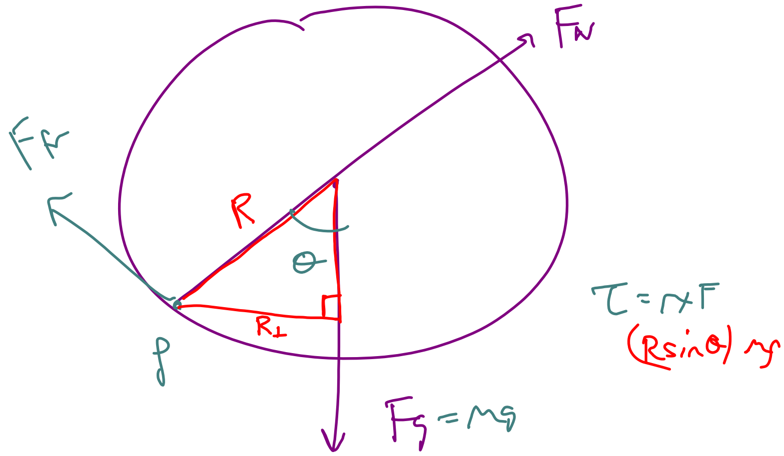
$$(\cancel{m} g \sin\theta)\cancel{R} = \left(\tfrac{1}{2}\cancel{M}\cancel{R^2} + \cancel{M}\cancel{R^2}\right)\left(\tfrac{a}{\cancel{R}}\right)$$

$$g\sin\theta = \frac{3}{2} a$$

$$\boxed{a = \frac{2}{3} g\sin\theta}$$

(b) $\sum F = ma$

$$\cancel{m} g\sin\theta - \mu(\cancel{m} g\cos\theta) = \cancel{m} a$$



(H) 1. $L = r \times p = I\omega$

2. $\Sigma \tau = \frac{dL}{dt}$, $\therefore$ $\Sigma \tau = 0 \Rightarrow L_1 = L_2$

(I) Gyroscope

(J) Statics : $\Sigma F = 0$, $\Sigma \tau = 0$