Let the downward direction be positive. We begin with $\sum \mathbf{F} = m\mathbf{a}$. As shown, the net force is $\sum \mathbf{F} = m\mathbf{g} - b\mathbf{v}$; thus, $m\mathbf{a} = m\mathbf{g} - b\mathbf{v}$ or $\mathbf{a} = \mathbf{g} - \frac{b}{m}\mathbf{v}$. Since $\mathbf{a} = \frac{d\mathbf{v}}{dt}$, we have the differential equation representing the object's motion while falling: $\frac{d\mathbf{v}}{dt} = \mathbf{g} - \frac{b}{m}\mathbf{v}$.

We wish to solve for $\mathbf{v}$ as a function of time. This equation is separable: $d\mathbf{v} = \left(\mathbf{g} - \frac{b}{m}\mathbf{v}\right)dt$. In order to maintain my sanity, I will now begin ignoring vectors. This equation then becomes $\dfrac{dv}{g - \frac{b}{m}v} = dt$. We are now ready to integrate; we will choose our limits as $t = 0$ to $t = t_1$ and $v = v_0$ to $v = v(t_1)$.

This gives $\displaystyle\int_{v_0}^{v(t_1)} \frac{dv}{g - \frac{b}{m}v} = \int_0^{t_1} dt$. An antiderivative for the left side is $-\dfrac{m}{b}\ln\left(g - \dfrac{b}{m}v\right)$; for the right side, t. Therefore, the equation is $-\dfrac{m}{b}\ln\left(g - \dfrac{b}{m}v\right)\Bigg]_{v_0}^{v(t_1)} = t\Big]_0^{t_1}$.

Plugging in the limits and subtracting, $-\dfrac{m}{b}\left(\ln\left(g - \dfrac{b}{m}v(t_1)\right) - \ln\left(g - \dfrac{b}{m}v_0\right)\right) - = t_1$. Logarithm manipulation rules allow the left side to be written as $-\dfrac{m}{b}\ln\dfrac{g - \frac{b}{m}v(t_1)}{g - \frac{b}{m}v_0}$. Now we can solve for $v(t_1)$: multiply both sides by $-\dfrac{b}{m}$, giving $\ln\dfrac{g - \frac{b}{m}v(t_1)}{g - \frac{b}{m}v_0} - = -\dfrac{b}{m}t_1$.

Then we raise e to the power of both sides to give $\dfrac{g - \frac{b}{m}v(t_1)}{g - \frac{b}{m}v_0} - = e^{-\frac{b}{m}t_1}$; multiplying through by the left side's denominator leaves $g - \frac{b}{m}v(t_1) - = e^{-\frac{b}{m}t_1}\left(g - \frac{b}{m}v_0\right)$, and then it's trivial to rearrange things to get $\frac{m}{b}\left(g - e^{-\frac{b}{m}t_1}\left(g - \frac{b}{m}v_0\right)\right) = v(t_1)$.

Frequently we assume $v_0 = 0$. If so, then $v(t_1) = \frac{m}{b}g\left(1 - e^{-\frac{b}{m}t_1}\right)$. The general shape of this graph is shown below, where the horizontal asymptote represents the terminal velocity $v_T = \dfrac{m}{b}g$.